

Name: _____

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HW 4.7 Advanced Trigonometric Identities: Sum and Product Identities:**PRODUCT TO SUM IDENTITIES**

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\cos a \sin b = \frac{1}{2} [\sin(a+b) - \sin(a-b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \sin b = -\frac{1}{2} [\cos(a+b) - \cos(a-b)]$$

SUM TO PRODUCT IDENTITIES

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

1. Evaluate each of the following using the formulas above: (No Calculators)

a) $2 \sin 45^\circ \cos 15^\circ$	b) $2 \sin 82.5^\circ \cos 37.5^\circ$
c) $2 \sin 127.5^\circ \sin 97.5^\circ$	d) $\cos 15^\circ$
e) $\sin 105^\circ + \sin 15^\circ$	f) $\cos 465^\circ + \cos 165^\circ$
g) $\sin 75^\circ - \sin 15^\circ$	h) $\frac{\cos 75^\circ + \cos 15^\circ}{\sin 75^\circ - \sin 15^\circ}$

2. Match each of the following sums from the left with the corresponding product on the right:

a) $\sin 4x + \sin 2x$		i) $2 \cos 4x \cos 2x$
b) $\sin 7x - \sin 3x$		ii) $2 \sin 35^\circ \cos 15^\circ$
c) $\cos 6x + \cos 2x$		iii) $-2 \sin 75^\circ \sin 5^\circ$
d) $\cos \frac{3x}{2} - \cos \frac{9x}{2}$		iv) $2 \cos 5x \sin 2x$
e) $\cos 65^\circ + \cos 15^\circ$		v) $2 \cos 55^\circ \sin 20^\circ$
f) $\sin 50^\circ + \sin 20^\circ$		vi) $2 \sin 3x \cos x$
g) $\cos 80^\circ - \cos 70^\circ$		vii) $2 \cos 40^\circ \cos 25^\circ$
h) $\sin 75^\circ - \sin 35^\circ$		viii) $2 \sin 3x \sin \frac{3x}{2}$

3. Prove the following Identities:

a) $\frac{\sin a + \sin 3a}{\cos a + \cos 3a} = \tan 2a$	b) $\frac{\sin 2a + \sin 4a}{\cos 2a + \cos 4a} = \tan 3a$
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$$\text{c) } \frac{\sin a + \sin b}{\sin a - \sin b} = \frac{\tan \frac{1}{2}(a+b)}{\tan \frac{1}{2}(a-b)}$$

$$\text{d) } \frac{\cos a + \cos b}{\cos a - \cos b} = -\cot \frac{1}{2}(a-b) \cot \frac{1}{2}(a+b)$$

$$\text{e) } \cos a + \cos 2a + \cos 3a = \cos 2a(1 + \cos a)$$

$$\text{f) } \frac{\sin 3a + \sin 5a + \sin 7a + \sin 9a}{\cos 3a + \cos 5a + \cos 7a + \cos 9a} = \tan 6a$$

4. Prove or Evaluate each of the following:

$(\cos 20^\circ)(\cos 40^\circ)(\cos 80^\circ) = \frac{1}{8}$	$\frac{(\cos 4x - \cos 2x)}{2 \sin 3x} = -\sin x \quad [\sin 3x \neq 0]$
$\cos 130^\circ + \cos 110^\circ + \cos 10^\circ = 0$	$\cos 220^\circ + \cos 100^\circ + \cos 20^\circ = 0$

Prove the following Identity:

$$1 + \cos 2\theta + \cos 4\theta + \cos 6\theta = 4 \cos \theta \cos 2\theta \cos 3\theta$$

5. Challenge: Evaluate the following using trig. identities:

$$\frac{\sin 13^\circ + \sin 47^\circ + \sin 73^\circ + \sin 107^\circ}{\cos 17^\circ}$$